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Microscopic derivation of the hydrodynamic equations for the superfluid fermi-systems

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Starting from the first principles of statistical mechanics the two-fluid hydrodynamics of superconductor in ideal approximation is constructed. For construction of hydrodynamics the system of equation of motion for a normal and an anomalous correlation functions are used. The transition to equations of hydrodynamics is execute with help an expansion of equations of motion for correlation functions in terms of a small parameter.

Key words: *Two-fluid hydrodynamics, correlation function, superfluid fermi-systems*

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1. Introduction

Owing to especial properties the quantum liquids are very interesting investigation object in physics. To them belong a superfluid He-3 and He-4, a superconducting fermi-systems, a trapped Bose gas, a core of neutron stars, etc [1,2]. In the weakly-nonequilibrium states the equations of hydrodynamics are very important instrument of description of the aforesaid systems. As known, the phenomena of superconductivity and superfluidity are deeply family. Property of the non dissipative current states, that arises up as a result of the phase transition to more-organized state, unites them [3]. That is not surprisingly, that equations of hydrodynamics of these systems have a similar structure (so called two-fluid equations). The phenomenological hydrodynamics of superfluid He-4 was constructed by Landau in 1941 [4]. At the microscopic level these equations were derived by Bogoliubov in 1963 [5]. The two-fluid model of superconductor was derived in 1965 by Svidzynsky and Slusarev [6], and independently by Stephen [7]. The system of equations of motion for the normal and the anomalous correlation functions are the starting point for construction of hydrodynamic equations in works [5,6]. The transition to equations of hydrodynamics is execute with help an expansion of equations of motion for correlation functions in terms of a small parameter. That small parameter is introduced formally. It is a noticeable lack of these works. In the work [5] it is a so called "parameter of homogeneity". In the work [6] it is a Plank constant, which obviously it was possible to put equal unit.

In the present paper, where following chart of work [6], starting from the first principles of statistical mechanics we derived the equations of two-fluid hydrodynamics of superconductor in an ideal approximation. An electronic liquid in the superconductor is described in the four-fermion Hamiltonian, that describe a direct interactions between electrons. In terms of the Heisenberg equations we constructed the equations of motion for correlation functions. The approaching of the mean-field for breaking up of higher correlation functions is used. Writing equations of motion in a dimensionless form, it is succeeded to select a small parameter. It is equal to the attitude of the length of coherence toward the characteristic length which macroscopic quantities change on (as the mean number of particles, momentum, energy). The expansion in terms of this small parameter coincide with the expansion in terms of a gradients of the macroscopic quantities.

2. Equations of motion for correlation functions

Let us consider a superconductor in the BCS model. In this model Hamiltonian of the system of electrons in the presence an external electromagnetic field in the second quantization representation has the next form (we set $\hbar = c = 1$ throughout this paper)

$$\hat{H} = \sum_{\sigma} \int d\vec{r} \Psi^{+}_{\sigma}(\vec{r}) \left\{ \frac{1}{2m} (\hat{\vec{p}} - e\vec{A}(\vec{r},t))^2 + eA_0(\vec{r},t) \right\} \Psi_{\sigma}(\vec{r}) + g \int d\vec{r} \Psi^{+}_{\uparrow}(\vec{r}) \Psi^{+}_{\downarrow}(\vec{r}) \Psi_{\downarrow}(\vec{r}) \Psi_{\uparrow}(\vec{r}). \tag{1}$$

For construction of the hydrodynamics of a systems with spontaneous broken symmetry we must to proceed from the expanded system of correlation functions [5]. To it come in besides a normal also an anomalous correlation function. Therefore will be issue a system correlation functions next form

$$\langle \Psi^{+}_{\uparrow}(x_1) \Psi_{\uparrow}(x_2) \rangle, \qquad \langle \Psi_{\downarrow}(x_1) \Psi_{\uparrow}(x_2) \rangle. \tag{2}$$

Here $x_i \equiv (\vec{r}_i, t)$, and dependence the creation and annihilation operators on the time is given through a Heisenberg representation, for instance $\Psi^{+}_{\uparrow}(x_1) = e^{i\hat{H}t} \Psi^{+}_{\uparrow}(\vec{r}_1) e^{-i\hat{H}t}$, the angular brackets indicate an average at the local-equilibrium ensemble.

Using an equation of motion of Heisenberg

$$i \frac{\partial \Psi_{\sigma}(x)}{\partial t} = \left[\Psi_{\sigma}(x), \hat{H} \right]_{-}, \tag{3}$$

we obtained the equations of motion for correlation functions (2). These equations has the form

$$\left\{ i \frac{\partial}{\partial t} + eA_0(x_1) - eA_0(x_2) \right\} \langle \Psi^{+}_{\uparrow}(x_1) \Psi_{\uparrow}(x_2) \rangle$$

$$+\frac{1}{2m}\left[(\hat{\vec{p}}_1 + e\vec{A}(x_1))^2 - (\hat{\vec{p}}_2 - e\vec{A}(x_2))^2\right]\langle\Psi_\uparrow^+(x_1)\Psi_\uparrow(x_2)\rangle \tag{4}$$
$$= -g\langle\Psi_\uparrow^+(x_1)\Psi_\downarrow^+(x_1)\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle + g\langle\Psi_\uparrow^+(x_1)\Psi_\downarrow^+(x_2)\Psi_\downarrow(x_2)\Psi_\uparrow(x_2)\rangle,$$

$$\left\{i\frac{\partial}{\partial t} - eA_0(x_1) - eA_0(x_2)\right\}\langle\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle$$
$$-\frac{1}{2m}\left[(\hat{\vec{p}}_1 - e\vec{A}(x_1))^2 + (\hat{\vec{p}}_2 - e\vec{A}(x_2))^2\right]\langle\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle \tag{5}$$
$$= -g\langle\Psi_\uparrow^+(x_1)\Psi_\downarrow(x_1)\Psi_\uparrow(x_1)\Psi_\uparrow(x_2)\rangle + g\langle\Psi_\downarrow(x_1)\Psi_\downarrow^+(x_2)\Psi_\downarrow(x_2)\Psi_\uparrow(x_2)\rangle.$$

The obtained equations of motion are unlocked because the correlation functions a higher order be a part of these equations. For closing equations we use the uncoupling of Hartree-Fock-Bogoliubov (the mean-field approximation). For instance

$$\langle\Psi_\uparrow^+(x_1)\Psi_\downarrow^+(x_1)\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle = \langle\Psi_\uparrow^+(x_1)\Psi_\downarrow^+(x_1)\rangle\langle\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle$$
$$+\langle\Psi_\uparrow^+(x_1)\Psi_\uparrow(x_2)\rangle\langle\Psi_\downarrow^+(x_1)\Psi_\downarrow(x_1)\rangle - \langle\Psi_\uparrow^+(x_1)\Psi_\downarrow(x_1)\rangle\langle\Psi_\downarrow^+(x_1)\Psi_\uparrow(x_2)\rangle. \tag{6}$$

And by analogy for the rest average. After using the uncoupling of type (6) the equation of motion (4), (5) will be has the form

$$\left\{i\frac{\partial}{\partial t} + eA_0(x_1) - eA_0(x_2)\right\}\langle\Psi_\uparrow^+(x_1)\Psi_\uparrow(x_2)\rangle$$
$$+\frac{1}{2m}\left[(\hat{\vec{p}}_1 + e\vec{A}(x_1))^2 - (\hat{\vec{p}}_2 - e\vec{A}(x_2))^2\right]\langle\Psi_\uparrow^+(x_1)\Psi_\uparrow(x_2)\rangle \tag{7}$$
$$= \Delta(x_2)\langle\Psi_\uparrow^+(x_1)\Psi_\downarrow^+(x_2)\rangle - \Delta^*(x_1)\langle\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle,$$

$$\left\{i\frac{\partial}{\partial t} - eA_0(x_1) - eA_0(x_2)\right\}\langle\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle$$
$$-\frac{1}{2m}\left[(\hat{\vec{p}}_1 - e\vec{A}(x_1))^2 + (\hat{\vec{p}}_2 - e\vec{A}(x_2))^2\right]\langle\Psi_\downarrow(x_1)\Psi_\uparrow(x_2)\rangle \tag{8}$$
$$= \Delta(x_1)\delta(\vec{r}_1 - \vec{r}_2) - \Delta(x_1)\langle\Psi_\uparrow^+(x_1)\Psi_\uparrow(x_2)\rangle - \Delta(x_2)\langle\Psi_\downarrow^+(x_2)\Psi_\downarrow(x_1)\rangle.$$

Here

$$\Delta(x) \equiv g\langle\Psi_\downarrow(x)\Psi_\uparrow(x)\rangle \tag{9}$$

is order parameter. We remark that in the obtaining equations (7), (8) the terms irrelevant with effect of superconductivity (terms of Hartree and Fock) are thrown out. However, taking into account the direct terms of Hartree are trivial, because they may be included in a potential $A_0(x)$.

The next step will be separation a gauge-noninvariant multipliers (in fact we pass to the frame of reference where the condensate is rest). Such separation of phase has the form [3]

$$\langle\Psi_\sigma^+(x_1)\Psi_\sigma(x_2)\rangle = \exp\{im(\chi(x_2) - \chi(x_1))\}\,G_\sigma(x_1, x_2),$$
$$\langle\Psi_\downarrow(x)\Psi_\uparrow(x)\rangle = \exp\{im(\chi(x_2) + \chi(x_1))\}\,F(x_1, x_2), \tag{10}$$
$$\Delta(x) = \exp\{2im\chi(x)\}\,|\Delta(x)|.$$

Then equations of motion for G and F are

$$\begin{aligned}\left\{i\frac{\partial}{\partial t}+\frac{1}{2m}\left[(\hat{\vec{p}}_1-m\vec{v}_s(x_1))^2-(\hat{\vec{p}}_2+m\vec{v}_s(x_2))^2\right]\right\}G_\uparrow(x_1,x_2)\\ +\left\{m\dot{\chi}(x_1)-m\dot{\chi}(x_2)+eA_0(x_1)-eA_0(x_2)\right\}G_\uparrow(x_1,x_2)\\ =|\Delta(x_2)|F^*(x_2,x_1)-|\Delta(x_1)|F(x_1,x_2),\end{aligned} \tag{11}$$

$$\begin{aligned}\left\{i\frac{\partial}{\partial t}-\frac{1}{2m}\left[(\hat{\vec{p}}_1+m\vec{v}_s(x_1))^2+(\hat{\vec{p}}_2+m\vec{v}_s(x_2))^2\right]\right\}F(x_1,x_2)\\ -\left\{m\dot{\chi}(x_1)+m\dot{\chi}(x_2)+eA_0(x_2)+eA_0(x_1)\right\}F(x_1,x_2)\\ =|\Delta(x_1)|\delta(\vec{r}_1-\vec{r}_2)-|\Delta(x_1)|G_\uparrow(x_1,x_2)-|\Delta(x_2)|G_\downarrow(x_2,x_1),\end{aligned} \tag{12}$$

where

$$\vec{v}_s=\nabla\chi-\frac{e}{m}\vec{A}$$

is superfluid velocity (velocity of the condensate). It is a gauge-invariant combination, because change of calibration may be compensate with help a gauge transformation a vector potential.

The transition to equations of hydrodynamics is execute with help an expansion of Eq. (11) and (12) in terms of a space gradients. This expansion a simply execute when we pass to so-called mixed Wigner representation [3]. For that we introduce a new varibles

$$\vec{R}=\frac{1}{2}(\vec{r}_1+\vec{r}_2),\quad \vec{r}=\vec{r}_2-\vec{r}_1.$$

After we doing the Fourier transformation with respect a relative coordinate $\vec{r}$. Therefore we obtain

$$f(x_1,x_2)\to f(\vec{R},\vec{r},t)=\int\frac{d\vec{p}}{(2\pi)^3}f(\vec{R},\vec{p},t)e^{i\vec{p}\vec{r}},$$

in addition

$$\begin{aligned}\vec{r}_1\to\vec{R}-\frac{i}{2}\nabla_{\vec{p}},\qquad \vec{r}_2\to\vec{R}+\frac{i}{2}\nabla_{\vec{p}},\\ \hat{\vec{p}}_1\to\vec{p}-\frac{i}{2}\nabla_{\vec{R}},\qquad \hat{\vec{p}}_2\to-\vec{p}-\frac{i}{2}\nabla_{\vec{R}}.\end{aligned} \tag{13}$$

Any function of $\vec{R}+\frac{i}{2}\nabla_{\vec{p}}$ can be understood in terms of its power-series expansion

$$f(\vec{R}+\frac{i}{2}\nabla_{\vec{p}})=f(\vec{R})+\frac{i}{2}\frac{\partial f(\vec{R})}{\partial\vec{R}}\frac{\partial}{\partial\vec{p}}-\cdots. \tag{14}$$

Using procedure (13) and (14) the equations for correlation functions can be written in the form

$$
\begin{aligned}
& i\left\{\frac{\partial}{\partial t}+eE_i\frac{\partial}{\partial p_i}-m\dot{v}_{si}\frac{\partial}{\partial p_i}-\frac{\partial}{\partial R_j}\left(\frac{(p_i+mv_{si})^2}{2m}\right)\frac{\partial}{\partial p_j}\right.\\
&\left.+\left(\frac{p_i}{m}+v_{si}\right)\frac{\partial}{\partial R_i}\right\}G_\uparrow(\vec{R},\vec{p},t)=|\Delta(\vec{R},t)|\left(F^*(\vec{R},\vec{p},t)-F(\vec{R},\vec{p},t)\right)\\
&-\frac{i}{2}\frac{\partial|\Delta(\vec{R},t)|}{\partial R_i}\frac{\partial}{\partial p_i}\left(F^*(\vec{R},\vec{p},t)+F(\vec{R},\vec{p},t)\right),
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&\left\{i\frac{\partial}{\partial t}-2eA_0(\vec{R},t)-2m\dot{\chi}(\vec{R},t)-i\left(p_i\frac{\partial v_{si}}{\partial R_j}\frac{\partial}{\partial p_j}-v_{si}\frac{\partial}{\partial R_i}\right)\right.\\
&\left.-\left(\frac{p^2}{m}+{v_s}^2\right)\right\}F(\vec{R},\vec{p},t)=|\Delta(\vec{R},t)|\left(1-G_\uparrow(\vec{R},\vec{p},t)-G_\downarrow(\vec{R},-\vec{p},t)\right)\\
&-\frac{i}{2}\frac{\partial|\Delta(\vec{R},t)|}{\partial R_i}\frac{\partial}{\partial p_i}(G_\uparrow(\vec{R},\vec{p},t)-G_\downarrow(\vec{R},-\vec{p},t)).
\end{aligned}
\tag{16}
$$

By obtaining these equations the second order terms with respect to space gradient (the terms proportional $\nabla^2_{\vec{R}}$) were neglected.

3. Two-fluid hydrodynamics

By solving equations (15), (16) we use the perturbation theory. Therefore, we must determine the order of other terms in these equations.

Let L is length which macroscopic quantities change on (it has order of the scale of system). Tnen

$$
|\nabla_{\vec{R}}|\sim L^{-1},\quad \frac{\partial}{\partial t}\sim\bar{u}L^{-1}\sim v_F\frac{T_c}{E_F}L^{-1},
\tag{17}
$$

where T_c is critical temperature, $E_F=\dfrac{mv_F^2}{2}=\dfrac{p_F^2}{2m}$ – Fermi energy. The characteristic momentums is order Fermi momentum, therefore we can put

$$
p=p_F+\frac{\xi}{v_F},\quad \xi\sim T_c.
$$

Then

$$
|\nabla_{\vec{p}}|\sim\frac{v_F}{T_c}\sim\xi_0\sim 10^{-4}\text{cm},
$$

where ξ_0 the coherence length. The gap Δ is order T_c.

This theory has two small parameter. It is ξ_0/L and $a/\xi_0=T_c/E_F$, where a – the interatomic distance. The first parameter associated with hydrodynamic approach, the second – with semi-classical. Further by semi-classical motion of electrons will be neglect. Therefore we may put $T_c\sim E_F$.

We denote

$$\alpha = \frac{\xi_0}{L} \ll 1. \tag{18}$$

Dividing Eq. (15) and (16) by the T_c and using (18) we get

$$\begin{aligned} i\alpha\Bigg\{ \frac{\partial}{\partial t} + eE_i \frac{\partial}{\partial p_i} - m\dot{v}_{si}\frac{\partial}{\partial p_i} - \frac{\partial}{\partial R_j}\left(\frac{(p_i + mv_{si})^2}{2m}\right)\frac{\partial}{\partial p_j} \\ + \left(\frac{p_i}{m} + v_{si}\right)\frac{\partial}{\partial R_i}\Bigg\} G_\uparrow(\vec{R},\vec{p},t) = |\Delta(\vec{R},t)|\left(F^*(\vec{R},\vec{p},t) - F(\vec{R},\vec{p},t)\right) \\ -\alpha\frac{i}{2}\frac{\partial|\Delta(\vec{R},t)|}{\partial R_i}\frac{\partial}{\partial p_i}\left(F^*(\vec{R},\vec{p},t) + F(\vec{R},\vec{p},t)\right), \end{aligned} \tag{19}$$

$$\begin{aligned} \Bigg\{ i\alpha\frac{\partial}{\partial t} - 2eA_0(\vec{R},t) - 2m\dot{\chi}(\vec{R},t) - i\alpha\left(p_i\frac{\partial v_{si}}{\partial R_j}\frac{\partial}{\partial p_j} - v_{si}\frac{\partial}{\partial R_i}\right) \\ - \left(\frac{p^2}{m} + {v_s}^2\right)\Bigg\} F(\vec{R},\vec{p},t) = |\Delta(\vec{R},t)|\left(1 - G_\uparrow(\vec{R},\vec{p},t) - G_\downarrow(\vec{R},-\vec{p},t)\right) \\ -\alpha\frac{i}{2}\frac{\partial|\Delta(\vec{R},t)|}{\partial R_i}\frac{\partial}{\partial p_i}(G_\uparrow(\vec{R},\vec{p},t) - G_\downarrow(\vec{R},-\vec{p},t)). \end{aligned} \tag{20}$$

In order to solve the equations (19) and (20) we formally expand the functions G, F and Δ in powers of α

$$f = f^{(0)} + \alpha f^{(1)}. \tag{21}$$

In the lowest order Eq. (19) gives

$${F^{(0)}}^*(\vec{R},\vec{p},t) = F^{(0)}(\vec{R},\vec{p},t), \tag{22}$$

that shows reality $F^{(0)}$. The Eq. (20) by using (22) in the lowest order gives

$$\begin{aligned} \left(eA_0(\vec{R},t) + m\dot{\chi}(\vec{R},t) + \frac{1}{2}mv_s^2(\vec{R},t)\right) F^{(0)}(\vec{R},\vec{p},t) \\ = -\frac{p^2}{2m}F^{(0)}(\vec{R},\vec{p},t) - \frac{1}{2}|\Delta^{(0)}(\vec{R},t)|(1 - G_\uparrow^{(0)}(\vec{R},\vec{p},t) - G_\downarrow^{(0)}(\vec{R},-\vec{p},t)). \end{aligned} \tag{23}$$

The separating variables gives

$$eA_0(\vec{R},t) + m\dot{\chi}(\vec{R},t) + \frac{1}{2}mv_s^2(\vec{R},t) + \mu(\vec{R},t) = 0, \tag{24}$$

and

$$2\xi_p F^{(0)}(\vec{R},\vec{p},t) + |\Delta^{(0)}(\vec{R},t)|(1 - G_\uparrow^{(0)}(\vec{R},\vec{p},t) - G_\downarrow^{(0)}(\vec{R},-\vec{p},t)), \tag{25}$$

where $\xi_p = \dfrac{p^2}{2m} - \mu$ and μ – is order separation of variables. In local equilibrium state μ – is chemical potential and Eq. (25) reduce to equation by order parameter.

Using operation of $\nabla_{\vec{R}}$ on Eq. (24) we gives equation of motion by superfluid velocity

$$m\frac{\partial\vec{v}_s}{\partial t}+\nabla_{\vec{R}}\left(\frac{m\vec{v}_s^2}{2}+\mu\right)=e\vec{E}. \tag{26}$$

By using the vector identity

$$\frac{1}{2}\nabla\vec{v}_s^2=\vec{v}_s\times(\nabla\times\vec{v}_s)+\vec{v}_s\cdot\nabla\vec{v}_s=-\frac{e}{m}\vec{v}_s\times\vec{\mathcal{H}}+\vec{v}_s\cdot\nabla\vec{v}_s.$$

Eq. (15) can be written in the form

$$m\frac{d\vec{v}_s}{dt}=e[\vec{E}+(\vec{v}_s\times\vec{\mathcal{H}})]-\nabla\mu. \tag{27}$$

This is the first hydrodynamical equation and shows that the superfluid accelerates freely under the applied fields. The remaining hydrodynamic equations are provided by the conservation relations for the particle density $\rho(\vec{R},t)$, momentum density $\vec{j}(\vec{R},t)$ and energy density $\mathcal{E}(\vec{R},t)$.

Let us consider the first order of equations (19) and (20). These equation has the form

$$\begin{aligned}&\frac{\partial G_{\uparrow}^{(0)}(\vec{R},\vec{p},t)}{\partial t}+\left(\frac{p_i}{m}+v_{si}\right)\frac{\partial G_{\uparrow}^{(0)}(\vec{R},\vec{p},t)}{\partial R_i}\\&+\frac{\partial G_{\uparrow}^{(0)}(\vec{R},\vec{p},t)}{\partial p_i}\left(eE_i-m\dot{v}_{si}-\frac{\partial}{\partial R_i}\left(\frac{(\vec{p}+m\vec{v}_s)^2}{2m}\right)\right)\\&-\frac{\partial\Delta^{(0)}(\vec{R},t)}{\partial R_i}\frac{\partial F^{(0)}(\vec{R},\vec{p},t)}{\partial p_i}=2|\Delta^{(0)}(\vec{R},t)|\mathrm{Im}F^{(1)}(\vec{R},\vec{p},t),\end{aligned} \tag{28}$$

$$\begin{aligned}&\frac{\partial F^{(0)}(\vec{R},\vec{p},t)}{\partial t}+2i\left(eA_0(\vec{R},t)+m\dot{\chi}(\vec{R},t)+\frac{1}{2}mv_s^2(\vec{R},t)+\frac{p^2}{2m}\right)F^{(1)}(\vec{R},\vec{p},t)\\&=\frac{\partial\Delta^{(0)}(\vec{R},t)}{\partial R_j}\frac{\partial}{\partial p_j}\left(G_{\uparrow}^{(0)}(\vec{R},\vec{p},t)-G_{\downarrow}^{(0)}(\vec{R},-\vec{p},t)\right)+\frac{\partial}{\partial p_j}\left(p_i\frac{\partial v_{si}}{\partial R_j}F^{(0)}(\vec{R},\vec{p},t)\right)\\&-\frac{\partial}{\partial R_i}\left(v_{si}F^{(0)}(\vec{R},\vec{p},t)\right)-i|\Delta^{(1)}(\vec{R},t)|\left(1-G_{\uparrow}^{(0)}(\vec{R},\vec{p},t)-G_{\downarrow}^{(0)}(\vec{R},-\vec{p},t)\right).\end{aligned} \tag{29}$$

The relations for ρ, $\vec{j}$ and $\mathcal{E}$ follow simply by taking moments of (28) and (29)(see Ref[6]).

From the definitions

$$\rho(\vec{R},t)=2m\int\frac{d\vec{p}}{(2\pi)^3}G_{\uparrow}^{(0)}(\vec{R},\vec{p},t). \tag{30}$$

After integrating (28) over $\vec{p}$ we find

$$\frac{\partial\rho}{\partial t}+\mathrm{div}\vec{j}=0, \tag{31}$$

where

$$\vec{j}(\vec{R},t) = 2\int \frac{d\vec{p}}{(2\pi)^3}\vec{p}G_{\uparrow}^{(0)}(\vec{R},\vec{p},t) + \rho\vec{v}_s \equiv \vec{j}_0 + \rho\vec{v}_s.$$

By analogy, a simple calculation gives

$$\frac{\partial j_k}{\partial t} + \frac{\partial \Pi_{ik}}{\partial R_i} = \frac{e}{m}\rho E_k + \frac{e}{mc}H_{ki}j_i, \tag{32}$$

where H_{ki} is magnetic field intensity tensor and stress tensor Π_{ik} is given by

$$\Pi_{ik} = \frac{2}{m}\int \frac{d\vec{p}}{(2\pi)^3}(p_i + mv_{si})(p_k + mv_{sk})G_{\uparrow}^{(0)}(\vec{R},\vec{p},t) + \delta_{ik}\frac{\Delta^{(0)}(\vec{R},t)}{g}. \tag{33}$$

The energy density (without mean-field energy) is

$$\mathcal{E}(\vec{R},t) = \frac{1}{m}\int \frac{d\vec{p}}{(2\pi)^3}(\vec{p}+m\vec{v}_s)^2 G_{\uparrow}^{(0)}(\vec{R},\vec{p},t) + \frac{|\Delta^{(0)}(\vec{R},t)|^2}{g}, \tag{34}$$

or

$$\mathcal{E} = \mathcal{E}_0 + \vec{j}_0\vec{v}_s + \frac{1}{2}\rho v_s^2,$$

where

$$\mathcal{E}_0 = \frac{1}{m}\int \frac{d\vec{p}}{(2\pi)^3}p^2 G_{\uparrow}^{(0)}(\vec{R},\vec{p},t) + \frac{|\Delta^{(0)}(\vec{R},t)|^2}{g}.$$

Using (28), (29) and (24) we obtained

$$\frac{\partial \mathcal{E}}{\partial t} + \mathrm{div}\vec{Q} = \frac{e}{m}\vec{E}\vec{j}, \tag{35}$$

the energy current is given by

$$\vec{Q} = \int \frac{d\vec{p}}{(2\pi)^3}(\vec{p}+m\vec{v}_s)(\frac{\vec{p}}{m} + \vec{v}_s)^2 G_{\uparrow}^{(0)}(\vec{R},\vec{p},t) + 2\frac{\Delta^{(0)}(\vec{R},t)}{g}\vec{v}_s. \tag{36}$$

The flows of hydrodynamic quantities may be calculated if we assume that as a zero approximation by the gradients be realized the thermodynamic local equilibrium. In local equilibrium

$$G_{\uparrow}^{(0)} = \upsilon_{\vec{p}}^2 + u_{\vec{p}}^2 f\left(\frac{\varepsilon_{\vec{p}} - \vec{u}\vec{p}}{T}\right) - \upsilon_{\vec{p}}^2 f\left(\frac{\varepsilon_{\vec{p}} + \vec{u}\vec{p}}{T}\right), \tag{37}$$

where

$$f(x) = (e^x+1)^{-1}, \quad u_{\vec{p}}^2 = \frac{1}{2}\left(1 + \frac{\xi_{\vec{p}}}{\varepsilon_{\vec{p}}}\right), \quad \upsilon_{\vec{p}}^2 = \frac{1}{2}\left(1 - \frac{\xi_{\vec{p}}}{\varepsilon_{\vec{p}}}\right), \quad \varepsilon_{\vec{p}} = \sqrt{\xi_{\vec{p}}^2 + \Delta^2}.$$

Using expression (37) we obtain

$$\begin{aligned} \vec{j} &= \rho_s \vec{v}_s + \rho_n \vec{v}_n, \quad \vec{v}_n \equiv \vec{u} + \vec{v}_s, \\ \Pi_{ik} &= \rho_n v_{ni} v_{nk} + \rho_s v_{si} v_{sk} + \delta_{ik} P, \\ \vec{Q} &= \left(\frac{v_s^2}{2} + \frac{\mu}{m} \right) \vec{j} + TS\vec{v}_n + \rho_n \vec{v}_n (\vec{v}_n \cdot (\vec{v}_n - \vec{v}_s)). \end{aligned} \tag{38}$$

Here

$$\rho_n = \frac{1}{u^2} \int \frac{d\vec{p}}{(2\pi)^3} \vec{p}\vec{u} f \left(\frac{\varepsilon_{\vec{p}} - \vec{u}\vec{p}}{T} \right)$$

is normal density,

$$\rho_s = \rho - \rho_n$$

is superfluid density,

$$P = TS - \mathcal{E}_0 + u^2 \rho_n + \rho\mu/m$$

is pressure, and entropy

$$S = 2 \int \frac{d\vec{p}}{(2\pi)^3} \ln \left(1 + \exp \left(\frac{\varepsilon_{\vec{p}} - \vec{u}\vec{p}}{T} \right) \right) + \frac{2}{T} \int \frac{d\vec{p}}{(2\pi)^3} f \left(\frac{\varepsilon_{\vec{p}} - \vec{u}\vec{p}}{T} \right).$$

The set of Eq. (27), (31), (32), (35) and (38) are form a full systems magneto-hydrodynamic equations for superconductor.

References

1. Nozières P., Pines D. The theory of Quantum Liquids. Addison-Wesley, Redwood City, California, 1990.
2. Yakovlev D.G., Levenfish K.P., Shibanov Yu.A., Usp. Fiz. Nauk, 1999, **169**, 825 (in Russian).
3. Svidzynsky A.V. Microscopic theory of superconductivity. Lutsk, Veza, 2001 (In Ukrainian).
4. Landau L.D., JETF, 1941, **11**, 592 (in Russian).
5. Bogoliubov N.N. To the question about hydrodynamics of superfluid liquid. Preprint UINR, Dubna, 1963 (in Russian).
6. Svidzynsky A.V. The space inhomogenous problems theory of superconductivity. Moscow, Nauka, 1982 (in Russian).
7. Stephen M.J. Phys. Rev. A, 1965, **139**, 197.

Мікроскопічний вивід рівнянь гідродинаміки для надплинних фермі систем

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Виходячи з перших принципів статистичної механіки побудовано дворідинну гідродинаміку надпровідника в ідеальному наближенні. Для побудови гідродинаміки використано систему рівнянь руху для нормальної та аномальної кореляційних функцій. Перехід до рівнянь гідродинаміки здійснюється через розклад рівнянь руху для кореляційних функцій за малим параметром.

Ключовү слова: Дворідинна гідродинаміка, кореляційна функція, надплинні фермі-системи

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