

Some problems of application of Volterra systems of kinetic equations and its expansion in nonlinear dynamics

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A systematic analysis of systems of kinetic population equations is presented, including Volterra and Lotka-Volterra. Population problems that had to be solved are investigated, and their brief analysis is provided. These problems include demographic, ecological, etc. problems. From a conceptual point of view, these problems are divided into two types: the problem of two species eating the same food (Volterra equation) and the predator-prey problem (Lotka-Volterra equation). The first problem arose from the problem of rabbit reproduction in Australia. In addition, in the same population biology, the problem arose when one species eats another (predator and prey). This problem was solved by many researchers in the field of biology and medicine, in particular virology. Its partial solution is given in the book of A. Lotka, and a more general one in the lectures of V. Volterra. Because of this, these equations are sometimes called the Lotka-Volterra equations. As in the first and second problems, it is necessary that there is enough resource (food) for the stationary stable existence and development of the dynamical system. We have analyzed the problems that are solved or that are expedient to be solved using these methods. Problems with a non-uniform temporal hierarchy of processes have also been analyzed. It has been shown that for solving such problems it is expedient to use the method of adiabatic elimination of variables. This method was used to solve kinetic problems in relaxation optics. These equations are expedient to use when there are several competing in-phase processes. Based on the general analysis of the systems of Volterra equations, it is possible to construct system criteria for controlling and predicting the corresponding processes and phenomena. To move to spatial problems, it is necessary to introduce the corresponding transport and diffusion coefficients into the systems of equations of Volterra and Lotka-Volterra. In this case, these equations can also be considered as systems of nonlinear diffusion equations. A list of problems for which it is expedient to use such a formalism is given.

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